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Fermonic zero-norm states and enlarged supersymmetries of Type II string

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Abstract. We calculate the NS–R fermionic zero-norm states of the type II string spectrum. The massless and some possible massive zero-norm states are seen to be responsible for the space-time supersymmetry. The existence of other fermionic massive zero-norm states with higher spinor–tensor indices correspond to new enlarged boson–fermion symmetries of the theory at high energy. We also discuss the R–R charges and R–R zero-norm states and justify the idea that the perturbative string does not carry the massless R–R charges.

1 Introduction

It has been known for a while that the complete space-time symmetry of string theory is closely related to the existence of zero-norm states [1] in the old covariant quantization of the string spectrum [2]. This includes the w_∞ symmetry of 2D Liouville theory and the *discrete* T duality symmetry of the closed bosonic string [3]. However, all previous studies were aimed at the bosonic sector and nothing has been said about the symmetries of the NS–R (Neveu–Schwarz–Ramond) and R–R sectors, ex. NS–R supersymmetry zero-norm states. On the other hand, a recent study has revealed that the D-brane is the symmetry charge carrier of the massless R–R state [4]. A further study from the zero-norm state point of view may hold the key to uncovering the whole set of R–R charges, including the massive ones.

In this paper, we will identify the fermionic zero-norm states which are responsible for the space-time supersymmetry in the NS–R sector of the type II string. This includes the massless and some possible massive ones since supersymmetry is an exact symmetry at each fixed mass level of the spectrum. For simplicity, we will calculate the zero-norm states up to the first massive level. In addition to the supersymmetry zero-norm states, we discover other massive zero-norm states with higher spinor–tensor indices which, presumably, correspond to new enlarged massive boson–fermion symmetries of the theory. We then calculate the massless R–R zero-norm states. It is found that these have exactly the same degree of freedom as those of the positive-norm physical propagating R–R gauge fields, and thus do not fit into the R–R charge degree of freedom. We conclude that the massless R–R zero-norm states *do not* correspond to the charges of massless R–R gauge fields. There are no perturbative string states which carry the massless R–R charges. This is very different from the NS–NS and NS–R sectors. A further study of the meaning of these massless R–R zero-norm states seems necessary.

This paper is organized as following. In Sect. 2, we calculate the NS zero-norm states including the first massive even G parity states. In Sect. 3, we calculate the massive R states and discover a type I massless fermionic zero-norm state and two types of massive zero-norm R states. Section 4 is devoted to a discussion of the physical meaning of all kinds of NS–R and R–R zero-norm states. In particular, supersymmetrical zero-norm states are identified.

2 Neveu–Schwarz states

In this section we work out all the physical states (including two types of bosonic zero-norm states) of the spectrum after GSO projection [5] in the NS sector of the open superstring. For the massless state (we use the notation in [1]),

$$\varepsilon_\mu b^\mu_{-1/2}\left|0,k\right\rangle ; \quad k\cdot\varepsilon=0. \tag{2.1}$$

The first massive states $(-k^2=m^2=2)$ are

$$\varepsilon_{\mu\nu\lambda}b^\mu_{-1/2}b^\nu_{-1/2}b^\lambda_{-1/2}\left|0,k\right\rangle ; \quad \varepsilon_{\mu\nu\lambda}\equiv\varepsilon_{[\mu\nu\lambda]},\ k^\mu\varepsilon_{\mu\nu\lambda}=0 \tag{2.2}$$

and

$$\varepsilon_{\mu\nu}\alpha^\mu_{-1}\alpha^\nu_{-1/2}\left|0,k\right\rangle ; \quad \varepsilon_{\mu\nu}=\varepsilon_{\nu\mu},\ k^\mu\varepsilon_{\mu\nu}=\eta^{\mu\nu}\varepsilon_{\mu\nu}=0. \tag{2.3}$$

In addition to the above positive-norm states, there are two types of zero-norm states in the NS sector. They are

Type I:

$$G_{-1/2}\left|\chi\right\rangle, \quad \text{where } G_{1/2}\left|\chi\right\rangle=G_{3/2}\left|\chi\right\rangle=L_0\left|\chi\right\rangle=0, \tag{2.4}$$

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and

Type II:

$$(G_{3/2} + 2G_{-1/2}L_{-1})\,|\chi\rangle\,, \quad \text{where } G_{1/2}\,|\chi\rangle = G_{3/2}\,|\chi\rangle = (L_0+1)\,|\chi\rangle = 0 \tag{2.5}$$

Note that Type I states have zero-norm at any spacetime dimension while Type II states have zero-norm only at $D = 10$. For the massless level, we have only one singlet Type I zero-norm state:

$$|\chi\rangle = |0,k\rangle\,, \quad -k^2 = m^2 = 0, \tag{2.6}$$

$$G_{-1/2}\,|\chi\rangle = k\cdot b_{-1/2}\,|0,k\rangle\,. \tag{2.7}$$

For the first massive level (even G parity, $-k^2 = m^2 = 2$)

Type I:

1.

$$|\chi\rangle = \theta_{\mu\nu} b^{\mu}_{-1/2} b^{\nu}_{-1/2}\,|0,k\rangle\,; \quad \theta_{\mu\nu} = -\theta_{\nu\mu}, k^{\mu}\theta_{\mu\nu} = 0, \tag{2.8}$$

$$G_{-1/2}\,|\chi\rangle = [2\theta_{\mu\nu}\alpha^{\mu}_{-1} b^{\nu}_{-1/2} + k_{[\lambda}\theta_{\mu\nu]} b^{\lambda}_{-1/2} b^{\mu}_{-1/2} b^{\nu}_{-1/2}]\,|0,k\rangle\,. \tag{2.9}$$

2.

$$|\chi\rangle = \theta_{\mu}\alpha^{\mu}_{-1}\,|0,k\rangle\,, \quad k\cdot\theta = 0, \tag{2.10}$$

$$G_{-1/2}\,|\chi\rangle = [\theta\cdot b_{-3/2} + (k\cdot b_{-1/2})(\theta\cdot\alpha_{-1})]\,|0,k\rangle \tag{2.11}$$

3.

$$|\chi\rangle = [\theta\cdot\alpha_{-1} + (k\cdot b_{-1/2})(\theta\cdot b_{-1/2})]\,|0,k\rangle\,, \quad k\cdot\theta = 0, \tag{2.12}$$

$$G_{-1/2}\,|\chi\rangle = \{2(k\cdot b_{-1/2})(\theta\cdot\alpha_{-1}) + (k\cdot\alpha_{-1})(\theta\cdot b_{-1/2}) + \theta\cdot b_{-3/2}\}\,|0,k\rangle\,. \tag{2.13}$$

The discovery of state (2.13) is suggested by noting that state (2.12) with $-k^2 = m^2 = 1$ corresponds to an odd G parity zero-norm state.

Type II:

$$|\chi\rangle = |0,k\rangle\,, \tag{2.14}$$

$$(G_{-3/2} + 2G_{-1/2}L_{-1})\,|\chi\rangle = \{3k\cdot b_{-3/2} + 2(k\cdot b_{-1/2})(k\cdot\alpha_{-1}) + (b_{-1/2}\cdot\alpha_{-1})\}\,|0,k\rangle\,. \tag{2.15}$$

The NS–NS sector of the Type II string spectrum can be obtained by

$$|\phi\rangle = |\phi\rangle_{\mathrm{R}} \otimes |\phi\rangle_{\mathrm{L}}\,, \tag{2.16}$$

where $|\phi\rangle$ is of zero norm if either $|\phi\rangle_{\mathrm{R}}$ or $|\phi\rangle_{\mathrm{L}}$ is of zero norm. As in the case of the bosonic string, each zero-norm state corresponds to a symmetry transformation on the bosonic sector.

3 Ramond states

We now discuss the interesting R sector. The zero-mode of the worldsheet fermionic operators $d^{\mu}_0 = (-\mathrm{i}/2^{1/2})\Gamma^{\mu}$, where Γ^{μ} obeys the 10D Dirac algebra

$$\{\Gamma^{\mu}, \Gamma^{\nu}\} = -2\eta^{\mu\nu}. \tag{3.1}$$

Define [6]

$$\Gamma^{a\pm} = \frac{1}{\sqrt{2}}(d^{2a}_0 \pm \mathrm{i}d^{2a+1}_0), \quad (a = 1,2,3,4); \qquad \Gamma^{0\pm} = \frac{1}{\sqrt{2}}(d^{1}_0 \mp d^{0}_0), \tag{3.2}$$

then

$$\{\Gamma^{a+}, \Gamma^{b-}\} = \delta^{ab}, \quad (a,b = 0,1,2,3,4). \tag{3.3}$$

That is, $\Gamma^{a\pm}$ are raising and lowering operators. The ground state of the R sector can be labeled by

$$\left|\vec{S}\right\rangle \equiv |S_0, S_1, S_2, S_3, S_4\rangle \equiv \left|\vec{S}, k\right\rangle u_{\vec{S}} \tag{3.4}$$

with $S_a = \pm 1/2$, $a = 0,1,2,3,4$, and $u_{\vec{S}}$ in (3.4) is the spin polarization. The 32 off-shell states decompose $32 \to 16 + 16'$ according to even (odd) numbers of -1/2 on S_a. The on-shell physical state conditions are

$$F_0 \left|\vec{S}, k\right\rangle u_{\vec{S}} = 0 \tag{3.5}$$

and

$$F_1 \left|\vec{S}, k\right\rangle u_{\vec{S}} = 0. \tag{3.6}$$

Equation (3.6) is automatically satisfied for the state in (3.4), and (3.5) implies the massless Dirac equation $k_{\mu}\Gamma^{\mu}_{\vec{S}',\vec{S}} \cdot u_{\vec{S}} = 0$. In the frame $k^0 = k^1, k^i = 0, i = 2,3,\ldots 9$, this implies $S_0 = 1/2$. The 16 on-shell states again decompose: $16 \to 8_s + 8_c \equiv u_{\vec{S}} + \overline{u}_{\vec{S}}$. At the massless level, the GSO operator reduces to the chirality operator, and only one of the chiral spinors 8_s (or 8_c) will be projected out. There are two massive vector–spinor states:

1.

$$\alpha^{\mu}_{-1} \left|\vec{S}, k\right\rangle u_{\mu,\vec{S}}. \tag{3.7}$$

The physical state conditions (3.5) and (3.6) give

$$[(k\cdot d_0)\alpha^{\mu}_{-1} + d^{\mu}_{-1}] \left|\vec{S}, k\right\rangle u_{\mu,\vec{S}} = 0, \tag{3.8}$$

and

$$d^{\mu}_0 \left|\vec{S}, k\right\rangle u_{\mu,\vec{S}} = 0, \tag{3.9}$$

2.

$$d^{\mu}_{-1} \left|\vec{S}, k\right\rangle \overline{u}_{\mu,\vec{S}}. \tag{3.10}$$

The physical state conditions (3.5) and (3.6) give

$$[(k\cdot d_0)d^{\mu}_{-1} + \alpha^{\mu}_{-1}] \left|\vec{S}, k\right\rangle \overline{u}_{\mu,\vec{S}} = 0, \tag{3.11}$$

and

$$k^\mu \left|\vec{S},k\right\rangle \overline{u}_{\mu,\vec{S}} = 0. \quad (3.12)$$

Note that (3.8) and (3.11) imply $-k^2 = m^2 = 2$ since $L_0 = F_0^2$.

In addition to the above positive-norm states, there are two types of zero-norm states in the R sector. They are
Type I:

$$F_0 \left|\psi\right\rangle, \quad \text{where } F_1 \left|\psi\right\rangle = L_0 \left|\psi\right\rangle = 0; \quad (3.13)$$

and
Type II:

$$F_0 F_{-1} \left|\psi\right\rangle, \quad \text{where } F_1 \left|\psi\right\rangle = (L_0+1)\left|\psi\right\rangle = 0. \quad (3.14)$$

We have used the superconformal algebra in the R sector to simplify the equations in (3.13) and (3.14). Note that, as in the NS sector, Type I states have zero norm at any space-time dimension while Type II states have zero norm only at $D = 10$. For the massless level, we have only one Type I zero-norm state

$$\left|\psi\right\rangle = \left|\vec{S},k\right\rangle \theta_{\vec{S}}, -k^2 = m^2 = 0, \quad (3.15)$$

$$F_0 \left|\vec{S},k\right\rangle \theta_{\vec{S}} = \frac{1}{\mathrm{i}\sqrt{2}} k_\mu \Gamma^\mu_{\vec{S}',\vec{S}} \left|\vec{S},k\right\rangle \theta_{\vec{S}}. \quad (3.16)$$

Note that $(k\cdot\Gamma)(k\cdot\Gamma)\left|\vec{S},k\right\rangle \theta_{\vec{S}} = 0$ implies, in the $k^0 = k^1$ frame, $S_0' = 1/2$. The spinor in (3.16) has 16 on-shell states as does the spinor in (3.4). In particular, the spinor $k\cdot\Gamma\left|\vec{S},k\right\rangle \theta_{\vec{S}}$ is left handed if $\left|\vec{S},k\right\rangle \theta_{\vec{S}}$ is right handed and vice versa. For the Type I massive level:

1.

$$\left|\psi\right\rangle = \alpha^\mu_{-1}\left|\vec{S},k\right\rangle \theta_{\mu,\vec{S}}, -k^2 = m^2 = 2, \quad (3.17)$$

$$F_1\left|\psi\right\rangle = d_0^\mu \left|\vec{S},k\right\rangle \theta_{\mu,\vec{S}} = 0, \quad (3.18)$$

$$F_0\left|\psi\right\rangle = [(k\cdot d_0)\alpha^\mu_{-1} + d^\mu_{-1}]\left|\vec{S},k\right\rangle \theta_{\mu,\vec{S}}. \quad (3.19)$$

2.

$$\left|\psi\right\rangle = d^\mu_{-1}\left|\vec{S},k\right\rangle \overline{\theta}_{\mu,\vec{S}}, -k^2 = m^2 = 2, \quad (3.20)$$

$$F_1\left|\psi\right\rangle = k^\mu \left|\vec{S},k\right\rangle \overline{\theta}_{\mu,\vec{S}} = 0, \quad (3.21)$$

$$F_0\left|\psi\right\rangle = [(k\cdot d_0)d^\mu_{-1} + \alpha^\mu_{-1}]\left|\vec{S},k\right\rangle \overline{\theta}_{\mu,\vec{S}}. \quad (3.22)$$

Equations (3.18) and (3.21) are gauge conditions. Note that (3.7), (3.10), (3.19) and (3.22) can be decomposed into $56_s + 8_c$ or $56_c + 8_s$ in the light-cone gauge. There is one Type II zero-norm state at this mass level;

$$\left|\psi\right\rangle = \left|\vec{S},k\right\rangle \theta_{\vec{S}}, \quad -k^2 = m^2 = 2, \quad (3.23)$$

$$F_0F_{-1}\left|\psi\right\rangle = [d_{-1}\cdot d_0 + k\cdot\alpha_{-1} + (k\cdot d_0)(k\cdot d_{-1}) + (k\cdot d_0)(\alpha_{-1}\cdot d_0)]\left|\vec{S},k\right\rangle \theta_{\vec{S}}. \quad (3.24)$$

4 Symmetry charges and zero-norm states of the NS–R sector

The fermionic zero-norm states can be obtained by (2.16), in which either $\left|\psi\right\rangle_{\mathrm{R}}$ ($\left|\psi\right\rangle_{\mathrm{L}}$) is an NS zero-norm state or $\left|\psi\right\rangle_{\mathrm{L}}$ ($\left|\psi\right\rangle_{\mathrm{R}}$) is an R zero-norm state. We first identify the massless supersymmetry zero-norm states which are responsible for the $N = 2$ massless space-time supersymmetry transformation. The obvious candidates are the products of the states in (2.7) and (3.4),

$$k\cdot b_{-1/2}\left|0,k\right\rangle \otimes \left|\vec{S},k\right\rangle u_{\vec{S}} \quad (4.1)$$

and

$$\left|\vec{S},k\right\rangle \overline{u}_{\vec{S}} \otimes k\cdot b_{-1/2}\left|0,k\right\rangle, \quad (4.2)$$

for the Type II A string. For II B string, the spinors in (4.1) and (4.2) are chosen to have the same chirality. Another possible choice is to use the zero-norm state in (3.16) on the R sector side, and either state in (2.1) or (2.7) on the NS sector side. For the case of (2.1), one notes that we have the tensor–spinor decomposition

$$8_v \otimes 8_c = 56_c \oplus 8_s, \quad (4.3)$$

$$8_v \otimes 8_s = 56_s \oplus 8_c, \quad (4.4)$$

where $56_{s,c}$ are gravitons. In this case, though we get the right supersymmetry spinor charge indices $8_{c,s}$, the $56_{s,c}$ zero-norm states do not correspond to any symmetry. One also notes that, as stated in the paragraph below (3.16), the state $k\cdot\Gamma\left|\vec{S},k\right\rangle \theta_{\vec{S}}$ has exactly the same degree of freedom as the positive-norm state $\left|\vec{S},k\right\rangle u_{\vec{S}}$ and also $u_{\vec{S}}, \overline{\theta}_{\vec{S}}$ have different chiralities. We conclude that the massless R zero-norm state (3.16) *does not* correspond to the space-time symmetry. A similar consideration will be discussed in the R–R sector at the end of this section.

We now discuss the massive zero-norm states. We first identify zero-norm states with only one space-time spinor index. They are all possible candidates for the massive space-time supersymmetry zero-norm states. For example, one can choose

$$(2.15) \otimes 8_{c,s}, \quad (4.5)$$

where (2.15) is the singlet Type II zero-norm state in (2.15), and $8_{c,s}$ are the components of the decomposition through (4.3) or (4.4) of any one of (3.7), (3.10), (3.19) and (3.22). Instead of using states (2.15) in (4.5), one can choose either the vector state in (2.11) or (2.13), and then make use of the decomposition (4.3) or (4.4) again to get the spinor $8_{c,s}$ states. We are unable to identify which states stated above do correspond to the massive supersymmetry transformation. However, the massive supersymmetry zero-norm states are surely among the set we stated above.

In addition to the massive spinor $8_{c,s}$ zero-norm states, we have many other fermionic massive zero-norm states with higher spinor–tensor indices which, presumably, will generate many new enlarged string-like massive boson–fermion supersymmetries. This is in analogy to the enlarged bosonic gauge symmetries in the NS–NS sector discussed in [2]. We give one example here:

$$\theta_{[\mu\nu]} \otimes 56_{c,s}, \tag{4.6}$$

where $\theta_{[\mu\nu]}$ is the zero-norm state in (2.9) and $56_{c,s}$ is the positive-norm state in either (3.7) or (3.10). Equation (4.6) represents the charge of a complicated inter-spin symmetry of string-like physics.

The massless R–R states of the Type II string consist of the antisymmetric tensor forms

$$G_{\alpha\beta} = \sum_{k=0}^{10} \frac{i^k}{k!} G_{\mu_1\mu_2\ldots\mu_k} (\Gamma^{\mu_1\mu_2\ldots\mu_k})_{\alpha\beta}, \tag{4.7}$$

where $\Gamma^{\mu_1\mu_2\ldots\mu_k}$ are the antisymmetric products of the gamma matrix, and α and β are spinor indices. There is a duality relation which reduces the number of independent tensor components to up to the $k = 5$ form. The on-shell conditions, or two massless Dirac equations, imply

$$k^{[\mu} G^{\nu_1\ldots\nu_k]} = k_\mu G^{\mu\nu_2\ldots\nu_k} = 0, \tag{4.8}$$

which means G is a *field strength* and can be written as a total derivative:

$$G_{(k)} = \mathrm{d}A_{(k-1)}. \tag{4.9}$$

Equation (4.9) has one important consequence, namely that perturbative string states do not carry the *massless* R–R symmetry charges. It was argued that the non-perturbative black P-branes [7] carry the R–R charges before Polchinski realized that D-branes [4] are exact string soliton states which carry the massless R–R charges. In the following , we will discuss this issue from the zero-norm state point of view. For the massless level, we have the following zero-norm state:

$$k_\mu \Gamma^\mu_{\vec{S}',\vec{S}} \left|\vec{S},k\right\rangle \theta_{\vec{S}} \otimes \left|\vec{S},k\right\rangle u_{\vec{S}}\ , \tag{4.10}$$

which decomposes again into tensor forms according to (4.7). Note that $k \cdot \Gamma\theta \otimes k \cdot \Gamma\theta = 0$. However, since the number of degrees of freedom of states in (4.10) does not fit into that of the R–R charges, and for reasons from the NS–R sector in the paragraph below (4.4), we conclude that zero-norm states in (4.10) do not correspond to symmetry charges. This justifies that the perturbative string does not carry the massless R–R charge.

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